The Hockeyplex Project

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1 Background

The sports page of a newspaper often makes statements such as "Team A has been mathematically eliminated from winning the division title." Such statements are often correct but usually based on simple rules. For example, suppose that, in a tournament, U of M has 27 wins and MSU has 22 wins, and each team has four games remaining. In this situation, it is certain that U of M will come out ahead of MSU even if they lose all remaining games. Even if MSU wins all of their remaining games they will only have 26 wins, one less than U of M already has. However, in a tournament with many teams, the outcome is not as easy to determine. The objective is to find a "good condition" that can be checked efficiently to identify if a team's outcome is already determined. Ideally, we also want to be able to efficiently convince others that our claim is correct. For example, if we want to claim that the Red Wings still have a chance to win the President's Trophy (that is, win the most points in a regular season), then all we have to do is to produce a possible outcome for all the remaining game that will give the Red Wings the most points. Convincing others that the statement "the Red Wings have been eliminated from winning the President's Trophy" is true, seems more difficult. It appears that we have to check all possible results of the remaining games. However, Hoffman [1960] showed that the above problem can be solved efficiently by using a branch of mathematics known as "network flow."

In large tournaments, many different methods are used to evaluate the certainty of teams qualifying for the finals, or winning the tournament. Many of these methods are based on simple rules, and can be weak (unable to discover factual outcomes) or inaccurate. Ribeiro and Urrutia [2003] cite an example of a 2002 football (soccer in the US) tournament in Brazil where the media declared São Paulo "mathematically qualified" for the final round. However, Riberiro and Urrutia were easily able to find possible outcome that left São Paulo out of the finals.

Although there is an efficient method to determine whether a team is "guaranteed mathematically" to win the President Trophy in the NHL, de-

termining whether a team has earned a position in the playoffs is difficult. Determining guaranteed playoff positions ahead of time belongs to a class of problems called *NP-Complete*. This is a class of problems that mathematicians and computer scientists know to be very difficult. (In fact, the Clay Mathematics Institute has offered a $1 million dollar prize to anyone who can come up with a "very good" method of solving problems in this class).

2 Project Description

The goal of our project was to apply a field of mathematics known as Integer Programming to solve the playoff problem for the NHL. Some of the research using this approach for MLB and a soccer league in Brazil include Robinson [1991], Alder, Erera, Hochbaum, Olinick [2002] and Ribeiro and Urrutia [2003]. These methods can not be applied directly to the NHL since the tournament is structured differently, but some ideas can be borrowed and modified.

In the NHL tournament there are 30 Teams, which are arranged in five team divisions. There are two conferences, the East and West, each containing 3 divisions. Teams are awarded two points for a win, one point for a tie, and zero points for a loss, and one point for an over time loss. After the regular season is completed eight teams are chosen from each conference to advance to the tournament. In each conference, qualifying teams are made up of the leader of each division, the team in each division with the most points, and the next five highest ranked teams in the conference according to score. Special rules are also employed to break ties between teams who have the same amount of points. The rules for the tournament are considerably more complicated than that of the Brazilian soccer tournament, in which there are no separate divisions and the qualifying teams are chosen by score alone.

In order to solve this problem we used a similar procedure to Ribeiro and Urrutia [2003], and developed a mixed integer program formulation for the following two problems: the Guaranteed Qualification Problem: is a given team guaranteed a place in the finals, and the Possible Qualification Problem: can a given team have a chance of qualifying. We developed software using C++ to generate the appropriate mixed integer program, and integrated our software with CPLEX©. In addition to solving the playoff qualification/elimination problem, we also modified our formulations to solve the Guaranteed Qualification and Possible Qualification problem for, Division Leader Status, Conference Leader Status, and Presidents Trophy.

In addition to the mathematical content of this project, the final stage of our project is publicizing our result to interested parties. This will include: building and maintaining a website with daily updates of standings and con-

tacting newspaper and television sports reporters to share our results. We will also be writing a complete technical paper detailing all of our work to be submitted to a journal.